# CHAPTER 4

## The consistency of analysis

### 4.1. Introduction.

We demonstrate a contradiction in the literature between Gentzen's completeness theorem, which uses infinite proofs to show that the axioms of arithmetic are consistent, and Godel's first and second incompleteness theorems, that an inspected system cannot be proved consistent within itself. The incompleteness theorems were derived by an extension of the 'diagonal argument' method used in *Superexponential algebra*, [Ad15], chapter XIV of the existence of undecidable problems in arithmetic, which we have deconstructed, and which we enlarged to these incompleteness theorems. We introduce the idea of forcing to prove that our version of the uncountable continuum hypothesis is independent of the other axioms of set theory. We then extend Gentzen's methods to prove the consistency of arithmetic. This is a proof whose stages can be defined by a two-valued (boolean) explosion.

To prove the consistency of analysis we will need to use our definitions of transcendental numbers. We have already defined transnatural and transalgebraic numbers. These numbers live in a system containing boolean logic and the extension of the proof of the consistency of arithmetic to the consistency of analysis using these two types of numbers can be found. However, if we define transcendental numbers as being inaccessible to transnatural and transalgebraic systems of proof, then there appears to be an obstruction to obtaining a proof by these methods, and there appears to be no method of computation that would provide it. The route to a solution can be found from the observation that transcendental numbers of this type satisfy formulas that are equi-provable in intuitionalistic logics. Since the latter have a computable realisation in colour logics, in which some values are hidden, we can provide a three-valued colour model containing these transcendental numbers, and then we can obtain a proof of the consistency of analysis, using these definitions in a three-valued explosion.

### 4.2. V = L.

### 4.3. Forcing.

### 4.4. Gentzen's consistency theorem for arithmetic.

A result of Gentzen states that using transfinite proofs, we can prove consistency of a system.

### 4.5. A Gentzen-type proof of the consistency of analysis.