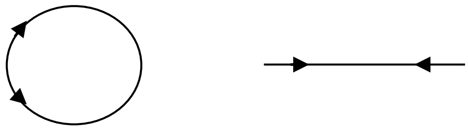
CHAPTER 3

Surgery

3.1. Introduction.

3.2. Surgery on unbranched spaces.

We will first look at the case of an unbranched space, where the admissible roots are x = 1, 2, -1 and -2, which correspond in one dimension to an oriented circle, an oriented interval, an unoriented circle and an unoriented interval respectively.



3.3. The idea of surgery on branched spaces.

By the fundamental theorem of algebra we know a complex polynomial equation in multiplicative format

$$(x + a)(x + b) \ldots (x + c) = 0 \qquad (1)$$

is uniquely expressed except up to order of factors, so this includes the case where a, b, ... , c are integer roots.

For a polynomial equation with integer coefficients in additive format

$$x^n + px^{n-1} + \ldots + q = 0 \qquad (2)$$

it may happen that (2) cannot generally be expressed in the form (1) when a, b, ... c in (1) are integers (but when (1) contains general roots, (1) and (2) map bijectively to one another).

Further, we have seen that the integer coefficients of (1) as a polynomial represent the Euler-Poincaré characteristic of a branched space.

The question then arises under what type of operations can a polynomial in additive format (2) with integer coefficients be converted to the form (1), and if the degree of the terms is maintained overall, can we define such an operation so that the conversion of (2) to (1) is unique in the general case? Moreover, if this is possible, what is the interpretation of such an operation?

We define such an operation as follows. Multiply out (1) so that it is in an additive form (2). We will assume that the leading coefficient of (1) and thus (2) is the number 1. Our interpretation of this case is that the polynomials refer to one branched space object. The case where there are m branched space objects then corresponds to a leading coefficient of m.

3.4. Surgery as additive maps.

3.5. Multiplicative maps.

3.2. Superstructural maps.