CHAPTER 1

Zargon lattices

1.1. Invitation to the reader.

The reader may find consulting the reference *Sphere packings, lattices and groups* (SPLAG) by J.H. Conway and N.J.A. Sloane [CS13] useful for this chapter.

Groups are the most investigated topic for superstructures with one operation. The theory of zargonions leads us in a natural way to ask what impact zargonions have on the theory of groups, even the classification of simple groups, for which the claim has been made that a complete classification was achieved after monumental efforts finally in 2008.

The application of the zargonion idea to these groups is a new subject for which there are currently few experts, and not initially the writer. Nevertheless, there are logical deductions which can be made, and I shall be making them. Since the conclusions do not tie in with what is believed to be a correct proof of the classification, which encompasses work by many individuals some of whom have diligently spent a lifetime on the study of simple groups, a question arises as to the source of this discrepancy. But it is our belief that the origin of this has now been isolated, which we describe to begin with informally in the next section.

The expert in group theory is invited to correct any misconceptions so that revisions can be made, and so the mathematician in the street can be better informed.

1.2. Introduction and history.

We introduce an analysis of group theory via the theory of novanions, also developed in [Ad15] and in the mathematical chapter XVII of *Investigations into universal physics* [Ad18b], by Graham Ennis and me. Adonion groups are found as a generalisation of the way octonions lead to special Lie groups. Using standard theory, we describe the way in which octonions lead to such groups and then extend these ideas to zargonion groups. In our initial attempt to study this subject without a proper understanding of its background, we looked at group structures derived from the nonassociative octonions. We noticed that the octonions have a (mod 12) Jacobi identity mapping to algebras, where octonions are related to some exceptional Lie algebras, like E_8, with corresponding mapping to groups. We investigate whether the standard approach and our own more limited methods give the same results here.

The beginning of this study arises out of a speculation of Daniel Hajas that I call the Hajas conjecture, in which novanions are connected with the heterotic string in physics, 10-novanions, to be described, are related to the 10-dimensional fermionic string in physics, and my extension of that idea, that the 26-dimensional bosonic string is related to 26-novanions. We will see we need to go beyond this. The heterotic string has a fermionic dimension of 10 and a bosonic dimension of 26, so that in the conventional picture 26 – 10 = 16 dimensions in the universe are compactified. We have shown in the above references that 26-novanions, containing octonion type algebras are also present.

We note that there are two distinct zargonion algebras of dimension 26, the 26-adonions and the 26-novanions. The discrepancy is described by different types of triplet decomposition in volume I, 4.11 and 4.12.

Picking up the work of Borcherds linking the monster simple group to orbifolds in the 26 dimensions of the heterotic string, we investigate whether there are simple groups beyond the monster, in direct conflict with current understandings. Our motivation is as follows. On page 938 of 'Quantum field theory' by Eberhard Zeidler, with reference to the Thompson series, he writes

"Borcherds calculated this series using the monster Lie algebra. This Lie algebra is constructed as the space of physical states of a bosonic string moving in a $\mathbb{Z}_2$ orbifold $M/\mathbb{Z}_2$ of a 26-dimensional torus M".

By this means the classification of simple groups is derived. Preliminary investigation of the work of Borcherds led credence to the Hajas conjecture, but subsequent more detailed investigations using (mod 12) algebra give mismatches in the computed size, called the order, of simple groups derived from novanions by our own methods. According to conventional wisdom this classification is a finite one. We wish to investigate whether or not a spanner can be thrown in the works, and note that the number of distinct n-novanionic algebras is infinite, given that the override condition in volume I sections 4.11 and 8.12 can always be allocated, which corresponds non-trivially when the n-novanion contains octonionic components. We will look at this proof of the finiteness condition and provide more in-depth investigations to compare standard reasoning with what is available from our own methods.

However, we note an interesting nonstandard conclusion we have arrived at with the Hajas identification, given essentially in [Ad18b] chapter XI, namely that all n-novanions can be allocated bosonic and fermionic parts. Thus both the 10- and 26-novanions may be allocated these components, and this also holds in general for n-novanions.

In chapter XX of [Ad18b] we employ the 26-novanions in Heim theory extended to gluons and quarks. There exist other possible universes with $n > 26$. So the 26-novanions do indeed contain a bosonic algebra.

A conjecture is: can we apply the unbounded zargonion algebra to derive a Thompson series so that the number of simple groups is not bounded?

We make this statement because override conditions for novanions of dimension greater than 26 lead to simple groups via Jacobi identity mappings to algebras, since overrides mix different quaternions together, creating one overarching structure. However, E_8 has structure enclosed in Λ_{26} linked to that of the monster. Some of the simple groups we are about to investigate have a size greater than that of the monster.

Although simple groups of similar order of magnitude to the monster have been obtained by zargon methods, some of them differ. Moreover, there are sequences of simple zargon groups without termination, violating the finiteness condition on the classification of sporadic groups. The groups corresponding to zargon algebras are not initially simple, but quotients with zargon subalgebras lead to factor groups which are simple.

Recent investigation reveals the reason for why the Borcherds and zargon constructions differ. The monster can be derived from vertex operator algebras with infinite quadratic form. Novanion algebras sometimes have zero quadratic form for a zero scalar component, t. These zero forms are not obtained as diagonalised forms which can be obtained by multiplication of the novanion by its conjugate. The Lie algebra construction forbids algebras of zero quadratic form. The scalar $t = 0$ component is realised in the final novanion algebra in conjunction with other additive elements to form a composite element, but we can map $t = 0 \rightarrow t = \pm 1$, and so novanions are allowable objects in such algebra constructions, although this has not hitherto been recognised. From this point of view the mapping from 10-novanions to fermion strings and 26-novanions to bosonic strings used in the Borcherds construction is relevant.

I am unclear at this stage whether other aspects of my reasoning are a novelty. These ideas are used to derive zargon groups, and can equally be used for division algebras like the octonions. The groups corresponding to zargon algebras are not initially simple, but quotients with zargon subalgebras lead to factor groups which are simple.

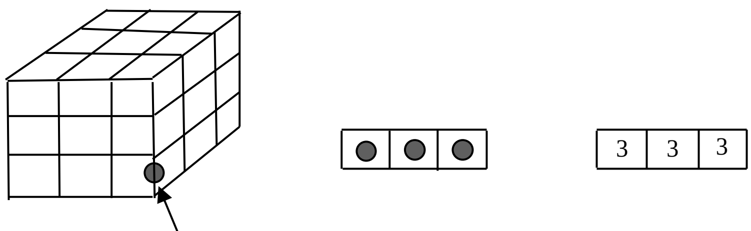
The development has proceeded in chapter 4 by first describing axiom systems which give mathematical structures for numbers and their generalisations. The intention is to provide the background for the discussion which follows. We discuss the classification of simple groups and their order. Various basic terms used in the classification such as the center of a group, factor groups, normal subgroups, Lagrange's subgroup theorem that the order of a subgroup divides the order of its group, and the Schur multiplier were introduced in volume I chapter 6. We derive various finite groups, which are not subgroups of the quaternions, escalated up from the quaternion Lie algebra.

We next give a detailed discussion of novanions, which are division algebras when the scalar part is not zero. Octonions are division algebras. We introduce 10-, 26- and more general zargonions. In the case of octonions, we show that they have a finite representation as a Lie algebra (mod 12), from which we may obtain a group. Products in the group which would otherwise be 0 (mod 12) can be converted to ± 1. This maintains the Lie algebra structure and means that the group derived from it is provided with inverses. Other zargonions may have no such Lie algebra structure, nevertheless a mapping of 0 (mod m) to ± 1 may still be made.

1.3. Diagrams.

A horizontal, vertical or diagonal line in a zargon box, each containing triplets or singletons, corresponds to a (mod 12) Lie algebra or quotients of (mod 12). However, more complicated interlacing of triplets is possible for adonion components than for novanion components, which always have standard closed triplets. The Lie algebra structure remains, but sets of elements in a triplet may not be closed under the algebra.

We give detailed zargon box diagrams in the manner of volume I, 4.14, for the structures we will encounter in this volume.



anchor node 3 singletons 3 triples

We note that for any (2 + 6k)-adonion, we can go along from an anchor node, and inside a 3-cube find two triples along a diagonal line. Then, for this adonion, we can choose a cube of sufficiently high dimension so that there is the anchor node and 6k triples in its interior. Thus any adonion of dimension (2 + 6k), can be embedded in a cube of dimension equal or greater than 3k, and certainly in an enveloping cube of infinite dimension 3**Ω**. This is the enveloping zargonion.

If we take a sheet of one face for the cube diagram on the left, and populate each square in the face with a triple like the rightmost diagram, then we have 27 – 2 = 25 nodes. The Lorentzian Leech lattice, $\Pi_{25,1}$ has 25 space nodes and one time node, and corresponds to this diagram. Vertically down the left hand side there are three sets of triples, corresponding to the space nodes of a 10-novanion with 9 space components (and one time component not shown in the diagram).

There is a reflection group connecting one space element of the zargon algebra with its one time component. If we ignore this pair, then from the Lorentzian Leech lattice we obtain the Leech lattice Λ_{24}, and from the 10-novanions correspondingly the lattice E_8. These are typical and important lattices used in the classification of simple groups.

We wish to enquire whether all simple groups can be constructed in this way. For instance a $\Pi_{19,1}$ Lorentzian adonion lattice has 19 space nodes and one time node. The 19 space nodes are constructed from three sets of six singletons plus the anchor node, where the six singletons plus the anchor node form the space components of an octonion, itself an adonion. Alternatively we can consider three octonions, each of which has seven space components, amalgamated together at an anchor node. This object appears not to be considered in the current classification of simple groups.

1.4. Tharlonions and quantum mechanics.

The algebra of tharlonions given in volume I, 4.18, does not include consistent division in its attributes, at least when we are not considering zero algebras. We will be confronted in works on physics where we will admit tharlonion structures in combination with tribbles and zargon algebras. The interpretation is that inconsistent physics which after a period of finite time self-annihilates, describes quantum mechanics from the group Heisenberg point of view, or else its Schrödinger abelianisation. Thus we always encounter timelike finite structures, but the zargon infinitely divisible space structures can remain.

The tharlonion equivalent of a zargon box now contains two copies of corresponding zargon boxes, except these special time components which exclude real number time all have square +1. Further, any two of these special time components multiply together to give a zargon space component.

1.5. For group theory, polynomial wheels are not relevant.

Polynomial wheel theory is ring theory outside the theory of groups. When we restrict ourselves to groups, the theory does not include polynomial wheels. For comparison methods, which add bogus roots to the solution, the situation is the same as subtraction in polynomial wheel theory. The classical Jordan-Hölder interpretation of solvability does not hold.