CHAPTER 4

Hamming and Golay codes

4.1. Introduction.

4.2. Hamming codes.

SPLAG Ch 3.

The E_8 lattice is very closely related to the (extended) Hamming code *H*(8,4) and can, in fact, be constructed from it. The Hamming code *H*(8,4) is a binary code of length 8 and rank 4; that is, it is a 4-dimensional subspace of the finite vector space $(\mathbf{F}_2)^8$. Writing elements of $(\mathbf{F}_2)^8$ as 8-bit integers in hexadecimal, the code *H*(8,4) can by given explicitly as the set

{00, 0F, 33, 3C, 55, 5A, 66, 69, 96, 99, A5, AA, C3, CC, F0, FF}.

The code *H*(8,4) is significant partly because it is a Type II self-dual code. It has a minimum Hamming weight 4, meaning that any two codewords differ by at least 4 bits. It is the largest length 8 binary code with this property.

One can construct a lattice Λ from a binary code *C* of length *n* by taking the set of all vectors *x* in $\mathbf{Z}^n$ such that *x* is congruent (modulo 2) to a codeword of *C*.[12] It is often convenient to rescale Λ by a factor of 1/√2,

Applying this construction a Type II self-dual code gives an even, unimodular lattice. In particular, applying it to the Hamming code *H*(8,4) gives an E_8 lattice. It is not entirely trivial, however, to find an explicit isomorphism between this lattice and the lattice Γ_8 defined above.

4.3. The Golay code.

References: p 649 SPLAG.

The binary Golay code, independently developed in 1949, is an application in coding theory. More specifically, it is an error-correcting code capable of correcting up to three errors in each 24-bit word, and detecting a fourth. It was used to communicate with the Voyager probes, as it is much more compact than the previously-used Hadamard code.

Quantizers, or analog-to-digital converters, can use lattices to minimise the average root-mean-square error. Most quantizers are based on the one-dimensional integer lattice, but using multi-dimensional lattices reduces the RMS error. The Leech lattice is a good solution to this problem, as the Voronoi cells have a low second moment.

The vertex algebra of the two-dimensional conformal field theory describing bosonic string theory, compactified on the 24-dimensional quotient torus $\mathbf{R}^{24}/\Lambda_{24}$ and orbifolded by a two-element reflection group, provides an explicit construction of the Griess algebra that has the monster group as its automorphism group. This **monster vertex algebra** was also used to prove the monstrous moonshine conjectures.